Determine ∇t at three different points, and assume that these vectors are independent. The three components of $\mathbf{v}$ can then be calculated by applying Eq. (2) at each of the listening stations. Note that the magnitude and direction of the speed of the object are thus obtained from Eq. (2) without using the locations of the listening stations.

Let $\mathbf{R}$ be a vector from the origin of coordinates to the vertex of the cone at time $t = 0$, and $\mathbf{L}$ a vector to a listening station. Then

$$\mathbf{L} = \mathbf{R} + \mathbf{v}t + \mathbf{S} \tag{3}$$

where $\mathbf{S}$ is an element of the cone extending from its vertex at time t to the listening station lying on the cone at that time. Thus,

$$\mathbf{R}\cdot\nabla t = \mathbf{L}\cdot\nabla t - t \tag{4}$$

The components of $\mathbf{R}$ may be calculated by solving simultaneously the three equations obtained by using Eq. (4) at each listening station.

This completes the determination of $\mathbf{v}$ and $\mathbf{R}$, and hence solves the problem of locating the object at any time. If in addition an explicit expression is desired for $\mathbf{v}$ or $\mathbf{R}$, it may be obtained in terms of the observed gradients by using the three cross products of them taken two at a time as a basis. By using Eq. (2), the velocity $\mathbf{v}$ is easily found in this way to be given by

$$\mathbf{v}[(\nabla t)_1\cdot(\nabla t)_2 \times (\nabla t)_3] = (\nabla t)_1 \times (\nabla t)_2 + (\nabla t)_2 \times (\nabla t)_3 + (\nabla t)_3 \times (\nabla t)_1 \tag{5}$$

A similar equation for $\mathbf{R}$ can be derived by applying Eq. (4) to get the coefficients of the cross products.

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Base Heat Transfer in an Axisymmetric Supersonic Flow

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THERE have been a number of theoretical and experimental studies of heat transfer in two-dimensional supersonic separated flows. Although some of these two-dimensional theoretical studies have been extended to the axisymmetric case,[1–3] few axisymmetric data are available for comparison. This Note presents the results of an experimental investigation of blunt-base heat transfer in an axisymmetric turbulent supersonic separated flow, and compares the results with published data and with recently published correlating parameters for both base heating and base cooling.

The experimental investigation was conducted in the Rutgers Axisymmetric Near-wake Tunnel (RANT), which is shown in Fig. 1. This vertical blow-down facility was designed specifically for the investigation of turbulent, supersonic near wakes of axisymmetric models in the absence of the usual support interference problems. The annular nozzle, incorporating a 3-in.-diam. cylindrical centerbody, was designed by the method of characteristics to produce a uniform flowfield at Mach 4.0. Models were attached to an appropriate centerbody extension, thus locating the model base at a position of optimum flow conditions. Detailed discussions of the construction and operation of the facility have been previously reported by Przirembel,[4] and Sieling, Przirembel, and Page.[5]

Steady-state stagnation pressure for RANT was maintained constant within $\pm\frac{1}{2}$ psia during each run, with run times of approximately 20 sec. Stagnation temperatures decreased at a rate of approximately 1°R/sec during each run. All tests in this investigation were conducted at a stagnation pressure of 152.2 psia and an average stagnation temperature of 510°R ± 10°R. The resulting freestream Reynolds number, 1.6×10^7 per ft, indicates a completely turbulent approaching boundary layer.

Three base-heat-transfer models were fabricated by using a steel collar and an appropriately machined Bakelite insert, as shown in Fig. 2. Thin copper heat meters, instrumented with 30 AWG copper-constantan thermocouples, were glued to the Bakelite insert. The blunt base of each model assembly was then polished to a uniform surface.

The average heat-transfer coefficient on each heat-meter surface was determined by a transient technique, utilizing an energy balance of the heat meter and assuming that the energy transfer was a quasi-steady process. An expression for the convective heat transfer then was derived in terms of the temperature-time gradient, the base temperature, and the tunnel stagnation temperature. The average heat-transfer coefficient for each heat meter was then obtained by measuring the instantaneous temperature and the rate of temperature change. A more detailed description of the model fabrication and experimental techniques employed has been reported previously.[6]

The average heat-transfer rates for a blunt base immersed in an axisymmetric flow were obtained with model HT 1. The heat-transfer rates measured with model HT 2 resulted in local heat-transfer coefficients situated at the coordinates of each heat meter, as shown in Fig. 2. The average heat-transfer coefficient for each heat meter then was considered a local heat-transfer coefficient located at a fixed point in the base region, and integrated over the base area to find the average heat-transfer coefficient for the base. Variation in heat-transfer coefficient due to radial change, or radial location, was measured with model HT 3.

Similar experimental heat-transfer coefficients were obtained with heat meters located 180° apart on model HT 2. This result, indicating a uniform angular heat-transfer rate, was a check on the axisymmetric flow conditions. Integration of these local values over the area of the base resulted in average heat-transfer values consistent with the results obtained with model HT 1.

The experimental heat-transfer coefficients obtained with model HT 3 indicated that the peak heating rate occurred

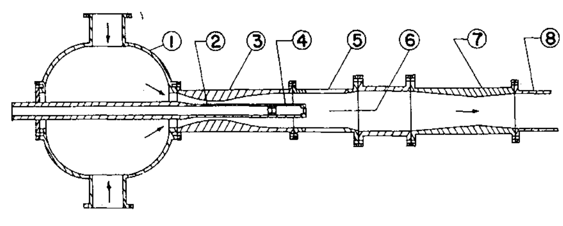

Fig. 1 Schematic of the Rutgers Axisymmetric Near-wake Tunnel (RANT): 1) settling chamber, 2) upstream sting, 3) nozzle, 4) model, 5) windows, 6) test section, 7) diffuser, 8) exhaust.

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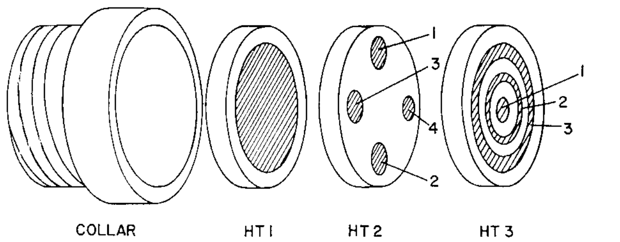

Fig. 2 Schematic of the blunt-base heat-transfer models.

at the model axis of symmetry and decreased to approximately three-fourths of this value as the edge of the model was approached. This variation in heat-transfer coefficient due to radial location was also found by Rabinowicz[7] and Swartz.[8] Integration of the data from these annular heat meters again resulted in average heat-transfer rates consistent with the results obtained with model HT 1. It should be pointed out that this average value was slightly less than the peak heating value measured at the base centerline; however, it was greater than the heat-transfer coefficient measured near the edge of the base.

The experimental investigations of Rabinowicz,[7] Swartz,[8] Bloom and Pallone,[9] Scott,[10] and Economos[11] were also conducted in a turbulent axisymmetric supersonic flow. Hence, it was possible to make comparisons between these data and results of the present investigation. It should be pointed out, however, that except for the investigations of Economos, and Bloom and Pallone, a side-mounted strut was used for model support. Such a support system generally causes interference effects, which cannot readily be corrected. Economos, and Bloom and Pallone, both utilized a model support system installed in the subsonic portion of the wind tunnel nozzle.

Although there are several techniques for the prediction and correlation of experimental base-heat-transfer data, the correlating parameters presented by Page and Dixon[12] appear to be the easiest to use with axisymmetric data. These correlating parameters, theoretically developed for two-dimensional conditions, depend upon the average base-heat-transfer rate, the stagnation temperature, the stagnation pressure, and the wall temperature-stagnation temperature ratio, with Mach number as an independent parameter. Curves of these correlation parameters are shown in Figs. 3 and 4. The two-dimensional experimental data of Naysmith[13] are shown in Fig. 3 to demonstrate the reliability of these correlation parameters.

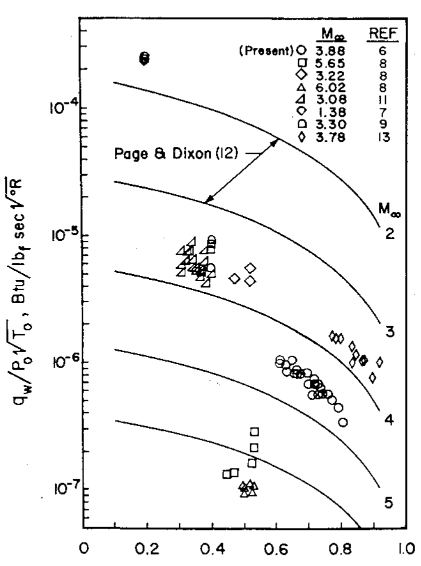

Fig. 3 Variation of the base-heat-transfer correlation parameter with temperature ratio ($T_w/T_0 < 1.0$).

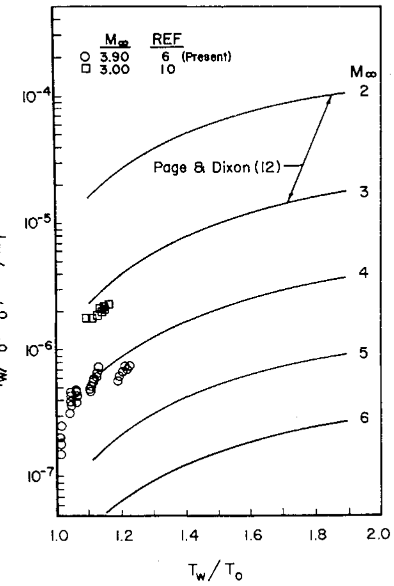

Fig. 4 Variation of the base-heat-transfer correlation parameter with temperature ratio ($T_w/T_0 > 1.0$).

Experimental results of Rabinowicz, Bloom and Pallone, Swartz, and Economos, as well as the data of the present investigation, are presented in Fig. 3 for comparison with the correlation curves of Page and Dixon. It should be pointed out that these data are all for conditions of a base cooler than the freestream stagnation temperature, a simulation of the proper direction of heat transfer for normal flight conditions. The axisymmetric data are slightly underpredicted for most cases. Note that the experimental data of the present investigation compare favorably with the correlation curves and that both magnitude and trend are adequately demonstrated.

The experimental results of Scott and the present investigation are compared with the correlation curves in Fig. 4. These data are for the case of the base temperature greater than the stagnation temperature. This corresponds to flight conditions in which drag reduction is accomplished by base heating. Again it is shown that the experimental data of the present investigation compare favorably with the correlation curves.

The results of the present investigation are in good agreement with the data of earlier investigators and tend to substantiate the available correlation curves.

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Hot-Wire Probes for Measuring Velocity and Concentration in Helium-Air Mixtures

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Introduction

THERE is a paucity of reliable data on fluctuating quantities in turbulent flows with variable density, principally because of the difficulties connected with making the appropriate measurements. One means of providing such data involves measuring with time-resolution the velocity and concentration of a foreign gas, e.g., of helium, in a low-speed isothermal flow with air as the second component. The mixing of ambient-temperature helium discharging at low speed from a circular orifice into quiescent air is an example of such a flow. By this means and by use of tape-recording techniques, the experimental difficulties, although formidable, would appear to be reduced and to be focused on the technique for sensing velocity and concentration.

One such technique, which we have been studying, is based on the suggestion of Corrsin[1] and involves two or more hot wires of sufficiently different characteristics so that from their instantaneous voltage signals one or more velocity components and the concentration of foreign gas can be inferred. Our purpose here is to report on the application of this technique to helium-air mixtures and in particular on a mode of hot-wire operation that appears to provide more promise than a conventional mode. We are interested in helium as the foreign gas because it makes feasible, with equipment available to us, the study of turbulent flows with significant density fluctuations.

We make several introductory remarks. To our knowledge, there have been three recent attempts to apply hot-wire anemometry to the measurement of velocity and concentration in turbulent flows of helium-air mixtures (cf., e.g., Tombach[2]). None have been very successful.‡ We describe our approach and results in terms of two wires used to determine one velocity component, say u, and the mass fraction of helium, denoted by c. Extension to a three-wire probe for measuring u, v, and c appears to be possible. Finally, we consider constant temperature operation with one sensor a wire and the second a film on a quartz fiber. Accordingly, we use the subscripts w and f to identify the two sensors.

Conventional Operation

The basic notion of the hot-wire technique for the present application is that the usual calibration based on King's law, i.e., on a voltage squared, $(u)^{1/2}$ relationship, must be extended to concentration so that the "calibration constants" become calibration functions of concentration. According to this notion, our two sensors follow the relations

$$\begin{aligned} E_w^2 &= A_w(c) + B_w(c)(u)^{1/2} \\ E_f^2 &= A_f(c) + B_f(c)(u)^{1/2} \end{aligned} \tag{1}$$

With the functions $A_w(c)$, $B_w(c)$, $A_f(c)$, and $B_f(c)$ known from calibration plus some curve-fitting of discrete data in c, then a pair of voltages, obtained during data collection, yield implicitly the concentration according to the following relation derivable from Eqs. (1):

$$E_w^2 = A_w[1 - (B_w/B_f)(A_f/A_w)] + (B_w/B_f)E_f^2 = a(c) + b(c)E_f^2 \tag{2}$$

With c determined from Eq. (2) either of Eqs. (1) yields u.

Now from King's law and the dependence of the fluid properties appearing therein on c, one can make estimates of the calibration functions for given sensors operating at given temperatures. Such estimates show that $a(c)$ is proportional to the thermal conductivity of the mixture and is thus a sensitive function of concentration. The calibration function b is essentially independent of concentration. However, we know from earlier work (cf. Refs. 3 and 4) that thermal slip effects due to the poor thermal accommodation of helium on most hot-wire materials makes suspect the application of King's law in helium-air mixtures. Thus experiment must be resorted to in order to assess the applicability of Eqs. (1) and (2) for present purposes.

In addition to establishing practical data on heat loss, sensitivity, etc. a certain amount of experimentation appears to be required in our probe development in order to obtain spatial resolution without interference between sensors; to achieve stable, reproducible performance from the sensors, etc. We report here on the results of a series of tests carried out on a wire-film combination consisting of a platinum wire of 0.0001 in. diam and 0.015 in. length and of a film of 0.001 in. diam with an active length of platinum of 0.010 in. The wire is unswept and is mounted in a plane orthogonal to the axis of the film at its center. The parameters that distinguish

Received January 9, 1970. This work was sponsored by the Air Force Office of Scientific Research, Office of Aerospace Research, U.S. Air Force, under Grant AF-AFOSR-927A-67. The results reported here relate to a project of larger scope, in the course of which the authors have benefited greatly from discussions with, and suggestions from, many people. We wish to explicitly acknowledge the help of J. LaRue in collecting the data presented here.

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‡ We have not experienced the "history" effect that is reported by Tombach and that caused him to abandon the hot-wire technique. However, in our mode of calibration and data collection, there is no occasion for us to expose the probe to high concentrations for long periods of time.